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ABSTRACT

A pretest-posttest control group design was used in a classroom experiment designed to determine whether the problem-solving performance of second grade students would be affected by systematic instruction in estimation. Participants were 36 students in two groups who were given daily lessons in measurement. The experimental group received a daily lesson in measurement mathematics that included estimation activities. The control group received daily mathematics lessons on measurement that included other types of measurement activities. Findings revealed a significant difference at the .05 level in the mean score differences between groups. It is concluded that second grade students benefit from measurement estimation lessons and activities. This type of instruction should be considered by teachers as a means of teaching estimation skills to second grade students. Appendices provide the pre- and posttest measures, 28 lesson plans for experimental and control groups, and statistical calculations. A total of 46 references are cited. (RH)

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THE EFFECTS OF INSTRUCTION IN ESTIMATION ON THE PROBLEM SOLVING PERFORMANCE OF SECOND GRADE STUDENTS

by

Lenell D. Sikes

A RESEARCH PROJECT REPORT

Presented in Fulfillment of the Requirements for the Specialist Field Project in the Graduate Program of Education at Mercer University in Atlanta

Atlanta, Georgia
1990

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OF SECOND GRADE STUDENTS

BY

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Purpose

The purpose of this study was to investigate the effect of systematic instruction of estimation in measurements on the problem solving ability of second grade students.

Methods and Procedures

Thirty-six second grade students from two intact classes were involved in the study. Both groups were given daily lessons in measurement. The control group was taught a measurement lesson which included an experiential activity. The experimental group was taught a measurement lesson which involved estimation in an experiential activity.

Results

The results of this study revealed a significant difference ($p < .05$) in the mean score differences between the experimental group and the control group.

Conclusions

The results of this study indicate that second grade students benefit from measurement estimation lessons and activities. This type of instruction should be considered by teachers as a means of teaching estimation skills to second grade students.

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Section I

INTRODUCTION AND PROBLEM STATEMENT

From ancient time to present day the use of estimation has gone through many phases. Famous mathematicians, such as Archimedes, estimated pi over 2000 years ago. Statistics is a prime example of the use of estimation since the 1900's. Mental arithmetic was an integral part of mathematics instruction in the middle of the 19th century. It was used as a form of exercise to improve mental discipline. Early in the 20th century movement away from emphasis on mental computation began as a strong negative reaction to the theory of mental discipline that prevailed during the latter part of the 19th century.

Mental discipline was revived in the 1930's and 1940's when the social usefulness of mathematics was emphasized. A deemphasis on mental computation occurred in the 1950's and 1960's when mathematics instruction began to focus on structural properties of the mathematical systems taught in schools. Methods of computation with approximate numbers were emphasized

superficially in the twelfth yearbook of the National Council of Teachers of Mathematics. In the 1980's the movement was back toward emphasis on mental computation. This was due to the "back-to-basics" movement and the use of technology in instruction (Reys, 1984).

Number sense can be described as good intuition about numbers and their relationships. Paper and pencil investigation of mathematics can only suggest ideas to be investigated. They cannot replace the "doing of mathematics" that is essential for the development of number sense (Howden, 1989). Making direct estimates of number is the most effective way of developing early number sense. According to Robert Reys (1989), second grade children need to develop a good number sense and be competent with basic arithmetic facts before doing computational estimation. Instruction in estimation should begin with an awareness of what estimation is about, so that students develop a tolerance for error (Reys, 1984).

Estimation needs to be taught in schools because mental computation and estimating are used in more than 80% of the real-world, problem solving situations outside the classroom (Reys & Reys, 1986). Estimation

skills are surprisingly poorly developed in students in the USA. They do not automatically develop with maturation nor from the study of more mathematics. Mental computation is not encouraged or suggested in most text books. Less than 5% of math lessons are devoted to computational skills (Reys, 1986). This fact suggests that much work needs to be done to bring about significant curriculum change.

A tremendous national effort is under way to bring about significant changes in the amount of instruction given to computational estimation, as well as to the way it is taught (Reys, 1984).

Many strategies may be used by good estimators. The most common strategy used is that of front end arithmetic, and the second most common strategy is rounding. Rounding is commonly taught in schools, but front end arithmetic is not (Poulter & Haylock, 1988). Estimation has usually been introduced in the third or fourth grades. Very little research has been done with estimation in early childhood education. In this experiential study, measurement lessons emphasizing estimation were taught to one class. This class constituted the experimental group. Measurement lessons

emphasizing other activities were taught to another class. This class constituted the control group.

Statement of the Problem

A review of the related literature indicates that adequate teaching of estimation is missing in the curriculum of American schools. The strategy of rounding is usually taught in third and fourth grade. Very little research in the area of estimation has been done below third grade. Robert Reys (1989) suggests that second grade students need to develop a good number sense and be competent with basic facts before doing computational estimation. The purpose of this study was to answer the following question: Is there a significant difference in the problem solving performance of second grade students who have systematic instruction in estimation?

Definitions

In order to provide a clearer understanding of the study, it is necessary to define the following terms.

Estimation: To form an approximate judgment or

opinion regarding the value, amount, size, weight, etc., of; to calculate approximately.

Mental Computation: The mental production of an exact answer performed without the use of a pencil and paper.

Front end strategy: A strategy in which estimators focus on the front end digits of the numbers involved.

Rounding: A strategy in which original numbers are rounded to a more manageable number that has the same number of digits.

Assumptions

The following are basic assumptions which should be considered in generalizing the results of this investigation.

1. The experimental and control groups would be equivalent.

2. Attendance would be stable through the experiment.

3. Students participating in this investigation would be equivalent in mathematical ability.

Limitations

The following are basic limitations which should be considered in generalizing the results of this investigation.

1. The study sample was limited to 36 second grade students between the ages of seven and eight years old.

2. The study lasted a period of five weeks.

3. The study was concerned only with students' measurement estimation skills and their effects on students' problem solving ability.

Section II

REVIEW OF LITERATURE

Within this section are the research findings in the field of estimation mathematics. Included in the review of the literature is the history of estimation mathematics. Also included is a review of the research that has been done with primary children in this area. A statement regarding the significance of the present study is also included.

Trends in Estimation

Since ancient time the use of estimation has gone through various phases. The following sections include discussions of the phases it has gone through to present day.

Ancient Time

Use of estimation dates back at least to ancient attempts to measure land, area, and time. Estimation has occupied the minds of such famous mathematicians as Archimedes, when he estimated pi over 2000 years ago, and Newton as he developed a method for

approximating solutions to equations in the seventeenth century (Usiskin & Bell, 1986).

Early Twentieth Century

A prime example of the use of estimation since the 1900's is statistics. Statistics is used to study populations through estimates based on samplings (Usiskin, 1986).

The twelfth yearbook of the National Council of Teachers of Mathematics was an early effort to focus attention on computational estimation. Methods of computation with approximate numbers were emphasized superficially (Reeve, 1937; Reys, 1984). Since 1900 work on computational estimation has appeared under the name of mental computation, approximations, or simply estimation (Buchanan, 1978).

Through the years mental computation has been a part of mathematics instruction. It has received varying degrees of attention, however. Mental arithmetic was an integral part of mathematics instruction in the middle of the 19th century. It was used as a form of exercise to improve mental discipline (Reys, 1984). Early in the 20th century, movement away from emphasis on mental computation began as a strong

negative reaction to the theory of mental discipline that prevailed during the latter part of the 19th century.

Middle Twentieth Century

Mental mathematics was revived in the 1930's and 1940's when the social usefulness of mathematics was emphasized. A deemphasis on mental computation occurred in the 1950's and 1960's when mathematics instruction began to focus on structural properties of the mathematical systems taught in schools.

Latter Twentieth Century

In the 1980's the movement is swinging back toward emphasis on mental computation. This movement is due to several things. One is the back to basics movement and the other is the use of technology and there recognition of the important role that mental computation plays in the efficient use of technology (Reys, 1984).

Developing Number Sense

Early Years

Number sense can be described as good intuition about numbers and their relationships. It develops

gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms. Since textbooks are limited to paper-and-pencil orientation, they can only suggest ideas to be investigated. They cannot replace the "doing of mathematics" that is essential for the development of number sense. No substitute exists for a skillful teacher and an environment that fosters curiosity and exploration at all grade levels (Howden, 1989).

Numerical estimation reflects high level cognitive processes that involve thinking and counting (Klahr & Wallace, 1976). One of the major developments of early childhood is the ability to understand regularity, invariance, and causality. There is a novelty and subjectivity in numerical estimation tasks that evoke children's mathematical inventiveness (Groen & Resnick, 1977; Resnick, 1980).

Children's ability to estimate increases with age. The reasons for this are: increased skill and strategy use in counting, experience with counting larger and larger quantities, and other varied experiences with numbers which provide children with more varied number sense.

Increased accuracy with estimation occurs with age because children tend to decrease information processing demands on themselves as a result of experience and understanding of numbers (Groen & Resnick, 1977; Klahr & Wallace, 1976; Siegler, 1981a).

Counting, estimating, and measuring are procedures that gradually become integrated into a child's body of quantitative knowledge (Hampel, 1985; Starkley & Gelman, 1982). Regardless of the child's age, greater counting skill is related to accuracy of estimation (Newman & Berger, 1984). A principal theme of kindergarten through eighth grade mathematics should be the effective use and understanding of number sense in applications as well as in other mathematical contexts (Conference Board of the Mathematical Sciences, 1982).

Hands on involvement in activities requiring estimation skills increases a kindergartener's math achievement and problem solving skills (Keller & Shanahan, 1983). In a research project at Indian Hollow Elementary School in Commack, New York, kindergarten students approach problem solving tasks with determination and skill using a robot named Big Track. Students have fascinating experiences as they

estimate directions, and distances in instructing Big Track to move to, and stop at, cardboard animal chairs. This hands on experience produces a sense of pride as students master their problem (Keller & Shanahan, 1983). The outcome of such activities is the development of divergent thinking, estimation skills, increased numerical concepts, decision making skills and development of directionality.

It is never too early for children to know that mathematics is not always exact and that a single "right answer" is not always attainable. Often a rough approximation is just fine. A person with number sense can size up a situation and decide whether an approximate number will do. Children should be taught about estimation and given many opportunities to make reasonable estimates (Newman & Turkel, 1983). Learning estimation skills may take place over a short period of time (Berger & Pintrich, 1986).

Primary Grades

Many young children develop informal methods of evaluating mentally well before they are taught pencil-and-paper methods in school. The early emphasis on written methods may halt the further

development of the ability to calculate mentally. The Soviets delay teaching children how to do written calculations until they become proficient with mental math (Reys & Reys, 1986). Tasks that involve estimation evoke children's mathematical inventiveness because it calls on them to think of methods other than traditional computational methods to solve problems (Groen & Resnick, 1977; Resnick, 1980).

As early as kindergarten, many opportunities exist or can be created to help children develop an experiential base from which better estimates can be made. As children are learning estimation skills, they are also developing a better sense of number. This can be facilitated by having children estimate an unknown quantity by: comparing it to a known quantity, partitioning it into known quantities, and using mental computation (NCTM, 1986).

It has been noted by the Office of Educational Research and Improvement (OERI) (1987) that children in kindergarten can correctly estimate relative sizes and proportions. A small number of experimental studies provide some evidence that instruction in estimation results in improved skill in solving

problems. The majority of these studies involve middle grade students.

Singer and Rita (1988) involve kindergarten children in using estimation skills by asking them to estimate how many groups of five blocks it will take to clean up the block corner. The children are involved in a hands on learning experience from which they gain many mathematical skills.

A personal letter was written directly to Robert Reys in April, 1989. In his response to that letter (July, 1989) he pointed out that second grade children need to develop a good number sense and be competent with basic facts before doing computational estimation. They should be able to do measurement types of estimation at this age. He suggests that instruction in estimation should begin with an awareness of what estimation is about, so that students develop a tolerance for error. Children need to develop a frame of reference which is essential to the development of number sense as well as the establishment of estimation skills.

Later Years

The lack of number sense is a prevailing factor

among otherwise sophisticated adults. These adults have only a vague understanding of numbers and succumb easily to arguments that refer to numbers they do not understand. Many have no appreciation of number magnitudes, no grasp of very large numbers, and little understanding of small ones. National reports have emphasized the need for number sense and the ability to use numbers in everyday living. The effective use and understanding of numbers in applications as well as other mathematical contests needs to be developed beginning in elementary school (Paulos, 1986; Turkel & Newman, 1988).

Thinking Skills

The major value of instruction and practice in mental computation is in helping students understand the flexibility and power of the number system. Estimation and mathematical thinking go hand in hand. Both are multifaceted, and each involves many different processes. Instruction in estimation provides a mental context not only to develop but also to practice many important thinking skills. The teaching of estimation can further some key thinking processes (R. Reys, 1987).

Making direct estimates of number is the most effective way of developing early number sense because it is the only way that is both direct and holistic. Only in direct estimates does the complete focus become the actual numbers rather than the algorithmic procedures (Reys & Reys, 1986).

Problem Solving

Problem solving is sometimes seen as something to be taught separately from other mathematical concepts, requiring its own alloted time slot, in an already crowded curriculum. However, problem solving can be used to help students develop an understanding of other concepts, thereby accomplishing two things at once. A problem should be presented that is interesting and requires the students to think and reason. Students should be allowed the opportunity to develop and extend their understanding of mathematics concepts and skills. Students should be given frequent opportunities to express their thinking in words to the teacher and to other students. This verbalization helps them formulate their own thoughts, clarify their ideas, get reactions from others, and hear others' points of view. Students should be allowed to use concrete materials

whenever appropriate, to help make mathematics concepts real and meaningful. This method of integrating problem solving with other concepts can be used with both early childhood students and older students (Burns & Van de Walle, 1987).

Technology

Studies have shown that the calculator can be used to improve all mathematical skills at all age levels (Fuys & Tischler, 1979; Leechford & Rice, 1982). Kennedy (1980) identified estimating as an activity for which children can use a calculator. The calculator can be a valuable instructional tool in the process of teaching for mastery learning.

Due to the increasing use of calculators and computers to do mathematical calculations, the use of good estimation skills is vital to test reasonableness of answers (R. Reys, 1984; Reys & Reys, 1987). It is easy to hit the wrong key when entering information into a calculator (B. Bestgen & R. Reys, 1982). The calculator gives the correct answer only if the correct keys are pushed in the correct order (Kennedy, 1980).

The widespread use of technology makes it necessary for school programs to reflect reality. As

technology relieves the need for cumbersome written computation, it increases the need for the wise and thoughtful use of technology. It also places a premium on good estimation skills.

In an interesting study by R. Reys, Bestgen, Rybolt, and Wyatt (1980), a calculator was wired to give incorrect answers. Students and adults who were good estimators were told to use the defective calculator to check their estimates. Even these good estimators, when confronted with the answer on the calculator, usually accepted it as correct and assumed their estimate was in error. This research suggests that students often place too much trust in a calculator's answer. Teachers should encourage their students to question the reasonableness of all answers, including those produced by a calculator or a computer.

Estimation

Who is a Good Estimator?

Good estimators have exceptional mental computational skills, but the converse is not true. People can be good at mental computation but not necessarily be good estimators. Students good at

estimation use many different strategies (Thompson & Walle, 1985).

Strategies

The most common strategy used is that of front end arithmetic. The second most common strategy is rounding. Front end arithmetic is rarely taught in schools. Rounding is conventionally taught in schools (National Institute of Education, 1975; Poulter & Haylock, 1988).

There are strategies that good estimators rely on that require experimentation, practice, and discussion. These strategies need to be practiced, discussed and experimented with. Regular sessions should be held. Two 10-minute sessions each week is a good idea. The sessions should not be too long (Bestgen & Reys, 1982).

Front end estimation is one strategy that should be taught. In this strategy, estimators focus on the front end digits of the numbers involved. This strategy is extremely effective when used for addition problems, but can be applied to other operations as well (Bestgen & Reys, 1982). Rounding is another strategy. In this strategy, estimators round the original numbers to a more manageable number that has

the same number of digits (Bestgen & Reys, 1982; NCTM, 1986).

In a comparable numbers strategy, estimators change the numbers in a given problem to numbers that are more manageable, yet close to the original digits. This strategy involves more than just rounding numbers. It requires a mental search for numbers that fit together and can be computed easily. Searching for compatible numbers is a particularly useful strategy for division problems, but it can be applied to other types of equations as well (Bestgen & Reys, 1982; NCTM, 1986).

The averaging strategy is a technique that does not lend itself to all types of problems and numbers. It is useful when all the numbers involved center on a particular value (Bestgen & Reys, 1982; NCTM, 1986).

Whatever strategies are used, students should always be encouraged to refine and compensate for their original estimates. This can be accomplished by stating whether an estimate is believed to be high or low, or by numerically adjusting the original estimate (Bestgen & Reys, 1982; NCTM, 1986).

As students experiment with estimation, they will probably develop strategies of their own. These should

be discussed with classmates. This will help them refine their strategy search for more effective techniques, and encourage others to try their ideas. Plenty of practice is very important (Bestgen & Reys, 1982; NCTM, 1986).

Estimation is a skill teachers cannot afford not to teach. It is a skill students will use the rest of their lives (Bestgen & Reys, 1982; NCTM, 1986; Office of Educational Research Instruction, 1987).

Appropriate materials must be developed to guide teachers and students in the acquisition of skills in mental computation and estimation. National Council of Teachers of Mathematics meetings must give more time to the topics of mental computation and estimation if teachers are to recognize their importance and more importantly, learn effective ways of teaching them. A better balance of forward looking topics such as estimation, problem solving, and technology must be reached (NCTM, 1986; B. Reys & R. Reys, 1986).

According to Reys and Reys (1986) and NCTM (1986), systematic instruction in computational estimation results in significant improvement in students' performance. It is clear that the teaching of

computational estimation strategies actually reinforces and stimulates good problem solving processes.

When to Use

Surveys show that mental computation and estimating are used in more than 80% of the real-world problem solving situations outside the classroom (Reys & Reys, 1986). Estimation is common to many settings and is associated with "real world" decisions. Estimations of quantity are typically made in situations where one does not have the time or inclination to count a large number of objects or where one is dealing with noncountable continuous quantities (Berger & Newman, 1984).

Many times an exact number is needed but other times an estimate will do just as well. Estimation produces an approximate answer (Reys & Reys, 1986).

Children need to realize that today's world is bombarded with numerical data. Some are exact and others are estimates. It is helpful and very important to know which is which. It is important to understand when exact answers are needed (R. Reys, 1986).

Mathematics is commonly seen as involving only exact ideas. Estimation does not seem to fit in all

this. Estimation seems to be just opposite to the nature and purposes of mathematics. Estimation is often thought of as a less attractive alternative to exact information. But estimating is often more reasonable than avoiding estimates and estimating is often the only choice one has in a situation. The uses of estimation fit the ideals of mathematics: clarity in thinking and discourse, facility in dealing with problems, and consistency in the application of procedures (Usiskin & Zalman, 1983).

Daily Use

Most people use estimation more often everyday than they do paper-and-pencil computation. Thus, computational estimation needs to be given more emphasis in school mathematics programs. Textbook analysis indicates that it has been one of the most neglected topics in the curriculum (Poulter & Haylock, 1985; Suydam, 1984).

Despite the importance of estimation in daily lives, few schools stress it. Schools that use estimation in teaching arithmetic do not introduce it soon enough (Office of Educational Research and Improvement, 1987).

Testing and Instruction

Performance on Assessments

According to Reys (1984), prior to 1984 computational estimation instruction was missing from the mathematics programs of schools. As a result of this, when the National Assessment of Educational Progress administered three mathematics assessments to different age groups over a 10 year span, the performance was low in this area. The conclusion was that estimation instruction was one of the most neglected skills in the mathematics curriculum (Carpenter, Colburn, Reys, & Wilson, 1976).

The National Council of Teachers of Mathematics did an assessment of student estimation skills. They found that at every age level, students performed at least 15 percentage points lower on estimation than on exercises involving exact computation (Bestgen & Reys, 1982).

Testing

The National Assessment of Education Progress found that in the USA estimation skills of students were very weak. Not many research studies have focused directly on computational estimation. Problems

involved in researching are that the process involved is difficult to identify. Criteria for good estimators is ambiguous and it is hard to separate from a students' computational ability (National Council of Teachers of Mathematics, 1986).

Robert Reys did a study in the USA which showed that estimation skills are surprisingly poorly developed and that they do not automatically develop with maturation nor from the study of more math. His study showed that students showed more proficiency when actually applying estimation skill, rather than doing straight computation (NCTM, 1986).

Data from tests of students' mental computation skills show that students' performance is low. The data generally reflect a lack of instructional attention (Reys & Reys, 1986). For years the only effort made to teach estimation has been that of teaching rounding. This has not been enough (Driscoll, 1981).

To teach more computational estimation means that more attention must be given to the systematic evaluation of estimation by classroom teachers and by standardized achievement tests. A review of several

standardized achievement tests showed that the coverage was sketchy and inadequate (McAloon & Robinson, 1988).

Valid measures of computational estimation can be obtained but require procedures different from conventional ones. Testing procedures should be updated to reflect current instructional strategies (McAloon & Robinson, 1988; Reys, 1984).

Schools should give special attention to testing computational estimation. Practical suggestions are: provide a small answer sheet that allows only space for answers, keep tests short (five to ten problems), use tests regularly, and control the time. Time can be controlled in several ways: use of overhead projector, use of slide projector, use of microcomputer, and use of tape recorder. Problems can be presented visually or auditorally for a few seconds. The teacher observes to see if students are using computation or not. Too much time is a test of computation. Too little time produces wild guessing. Somewhere between these two extremes will be an optimal time that can be often determined by observing students carefully (McAloon & Robinson, 1988; Reys, 1984).

We must do all we can to insure that testing estimation (from teacher tests to nationally

standardized achievement tests) is not only appropriately represented, but well assessed. It is becoming obvious that students need and can greatly benefit from some skill in mentally computing arithmetic problems. Students should not be limited to written computation but should take advantage of whatever type of computation is most efficient for a problem whether it be written, mental, or mechanical (B. Reys, 1985).

School programs tend to include those curriculum areas that students will encounter on standardized tests. Traditionally estimation problems have not been included on standardized achievement tests. With changes under way that will include estimation mathematics in school curriculums, it is important that standardized tests given in school systems include problems in estimation mathematics so that progress in schools will not be inhibited (R. Reys, 1984).

Absent From Curriculum

Mental computation is not encouraged or suggested in most textbooks. As much as 60 to 75% of lessons in current textbooks center on computational skills, but less than 5% is devoted to computational estimation

(Reys & Reys, 1986). These facts suggest that much work needs to be done to bring about significant curriculum change.

Curriculum Change

Estimation skills are essential and must be given high priority within every school program. Few mathematics topics provide the wealth of benefits, both immediate as well as long term, as does estimation (R. Reys, 1987).

Traditionally computational estimation has been introduced in about the fourth grade, but ignored in work with computation in higher grades. Up until 1984, a review of math basal series show little instruction devoted to developing estimation skills (Driscoll, 1981; R. Reys, 1984). When presented, it usually centered on rounding. In a survey of good estimators in 1984, most responded that they were taught to round numbers not to estimate (Reys, Bestgen, Rybolt, & Wyatt, 1980). The subjects interviewed did not know how they developed skills in estimation, but suggested that they learned it on their own because they needed it (R. Reys, 1984).

A tremendous national effort is now under way to

bring about significant changes in the total amount of instruction given to computational estimation, as well as to the way it is taught (R. Reys, 1984). A shift in emphasis must occur from a written computational curriculum to a more realistic balance among written, mental, exact, and approximate types of computation. Mental computation and estimation have always been viewed as important, but the attention to instruction and testing in these skills has long been dwarfed by a heavy emphasis on written algorithms (Reys & Reys, 1986).

Systematic Instruction

Students need practice in estimation because it is a skill that is used far more often than exact computation (Bestgen & Reys, 1982). Instruction in estimation provides a natural context not only to develop but also to practice many important thinking skills. The teaching of estimation can further develop problem solving skills (R. Reys, 1987).

Research shows that student performance in estimation will not improve automatically with age or with further general study in mathematics. Estimation must be taught specifically.

Students should be taught to think of estimation in a positive way. They should understand that estimation serves a quick and useful purpose when time is short and a quick decision is needed. In today's complicated world, quick decisions are a way of life and estimation skills are more important than ever before (Bestgen & Reys, 1982; Carpenter, 1976; National Council of Supervisors of Mathematics, 1977; National Council of Teachers of Mathematics, 1986; National Institute of Education, 1975).

Research

Since estimation has usually been introduced in the third or fourth grades, it is not surprising that very little research has been done in early childhood education. One exception is a study by Immers (1983) that included second graders. To determine the effect of estimation on instruction in the linear estimation ability and strategy use of elementary students, Immers employed a pretest-posttest control group design. Forty-eight students in each grade from grades two through five (n = 192) from one school were randomly assigned to either an experimental group or a control group. A 10-day microcomputer-based instructional unit

was prepared by the researcher. Immers reported that linear estimation ability correlated significantly with age for both treatment groups and that there were no significant sex differences. Instruction was the most effective for the two weakest pretest areas--long and vertical estimation. Instruction benefitted students in grades three through five but apparently not second graders. The experimental group performed significantly better on short and horizontal estimates and on the remaining subdivisions of the posttest than the control group (Benton, 1986).

Another experimental study (Schoen, Friesen, Jarrett, & Urbatsch, 1981) used subjects from two intact fourth-grade classes of 21 students each. The researchers developed five 45-minute estimation lessons designed to teach students to learn to estimate the products of one-digit and two-digit, one digit and three digit, and two-digit and two-digit factors. In a separate part of the study they also taught and tested fifth and sixth graders on similar estimation tasks. The researchers concluded that estimation in whole number computation can be taught in a short period of time. Not only fourth, fifth, and sixth graders become better estimators, they did so by adopting a

valid estimation strategy. These skills and strategies were retained for at least a three-week period (Benton, 1986).

Sowder and Wheeler (1989) found in their study, "The Development of Concepts and Strategies used in Computational Estimation," that careful development of foundational concepts is recommended to prevent learning computational estimation as a set of algorithmic rules. Subjects of their study were from grades three, five, seven, and nine. The heavy emphasis on computation in the curriculum within the span of these grade levels makes them likely ones in which to observe the development of concepts and skills associated with computational estimation. In Case's (1985) theory of intellectual development, students in grades three and five are at the final two substages of his Dimensional stage and students in grades seven and nine are at the first two substages of his Vectorial stage. Therefore, by selecting groups of students at these grade levels, it was possible to test at each relevant substage.

Significance of the Study

Little research has been devoted to the teaching

and learning of estimation. One reason involves the difficulty of testing estimation skills. It will be wise to continue attempts to develop and administer better instruments for assessing estimation abilities. More research is needed in the choice of strategies. That is, once strategies are classified (and further study and replication is needed in this area), then how and why does a person choose a certain strategy? The most pressing need is for experimental research of estimation and how it should be taught (Benton, 1986).

At all levels, studies are needed that address the important interrelationship between estimation and conceptual thinking (NCTM, 1986). In the early grades, it would be helpful to determine if students in grades one through three can be effectively taught some simple estimation skills. The purpose of this study therefore was to determine if second grade students can be taught to estimate in the area of measurement and what effect this instruction has on their problem solving ability.

Section III

METHODOLOGY

The purpose of this section is to describe the procedures used in the study. Two groups were involved in this study. One group was taught a daily lesson on measurement, including activities involving estimation. The other group was taught daily lessons on measurement including other kinds of measurement activities. In this section the hypothesis is stated and the testing instrument is presented and described. This section also includes the sampling technique, experimental design, and the data analysis.

Statement of the Hypothesis

This study was designed to test the following hypothesis at the .05 level of significance.

Ho: There is no significant difference in the problem solving performance on estimation tasks of second grade students who are taught measurement lessons including estimation activities and those who

are taught measurement lessons including other kinds of measurement activities.

Hr: There is a significant difference in the estimating ability of second grade students who are taught measurement lessons including estimation activities and those who are taught measurement lessons including other kinds of measurement activities.

Sampling Technique

Half of the students in the sample are from white middle-class families living in single family dwellings. Twenty-five percent are low middle class apartment dwellers. The remaining 25% are a mixture of Asian Americans and Black Americans. They reside in central Dekalb County, Georgia. The sample consisted of 36 students, 19 boys and 17 girls.

The study involved two intact classes. The control group consisted of nine boys and nine girls. The experimental group consisted of ten boys and eight girls. The control group will receive daily lessons in measurement mathematics including measurement activities. The experimental group will receive daily measurement mathematics lessons including estimation activities. The experiment lasted for 24 days.

Instrumentation

Before the study began, a pretest was administered. The test was designed by the investigator and consisted of 10 estimation problems to solve. The problems were presented one at a time on the overhead projector. The problem was read for the students and projected for 30 seconds. The test required the students to use estimation ability to solve measurement estimation problems. Each test item was scored within two of the exact answer. The tests were evaluated by two experts in the field of education for content validity. At the end of the study a similar posttest was administered in a similar fashion.

Experimental Design

The pretest-posttest control group design was selected for this study. The experimental group received a daily lesson in measurement mathematics that included estimation activities. The control group received daily math lessons on measurement mathematics that included other types of measurement activities.

The design of the study is shown in Table 1.

Table 1

A Pretest-Posttest Control Group Design

Group	n	Independent Variable	Pretest	Posttest
E	18	X	T_1	T_2
C	18	Y	T_1	T_2

E: Experimental Group

C: Control Group

n: Number of students

X: Measurement estimation

Y: No measurement estimation

T_1: Pretest

T_2: Posttest

Procedure

The experimental group was given lessons in measurement and were involved in an estimation experience daily for four weeks. The lessons were followed by four days of review using chapter nine in Book Two of the Addison-Wesley Mathematics series. The control group was given lessons in measurement mathematics including other kinds of activities daily for four weeks. The lessons were followed by four days

of review using pages in chapter nine in Book Two of the Addison-Wesley Mathematics series.

The experimental group received estimation lessons in the following kinds of measurement: capacity, mass, and linear measurement. Both groups were administered the same teacher made test for the pretest and the posttest. Detailed lesson plans are included in the appendix. (See Appendix B.)

Statistical Analysis

To test the hypothesis formulated for this experimental study, data was collected and tested by a *t* test for the difference between the pretest and posttest means for the control group and experimental group with a .05 level of significance.

Section IV

RESULTS AND CONCLUSIONS

The purpose of this study was to answer the following question: Is there a significant difference in the problem solving performance of second grade students who are given systematic instruction in estimation? A summary of the statistical results, observations, conclusions, limitations, and implications for future research and classroom practice are reported here.

Results

The hypothesis was formulated and tested at the .05 level of significance. The null hypothesis stated there was no difference in the problem solving performance on estimation tasks of second grade students who are taught measurement lessons including estimation activities and those who are taught measurement lessons including other kinds of measurement activities.

The research hypothesis stated that there was a significant difference in the problem solving performance of second grade students who are taught measurement lessons including estimation activities and those who are taught measurement lessons, including other kinds of measurement activities.

The hypothesis was tested by investigator developed instruments. The 36 members of the sample were from two intact classes. The control group received daily measurement lessons including experiential measurement activities. The experimental group received daily measurement lessons including experiential measurement activities that involved estimation skills. The experiment lasted 24 days. The children were given a pretest and a posttest means. Data was collected and tested by a t test for a difference between the group means. The statistical results are reported in Table 2.

As noted in Table 2, the t score for the difference between pretest and posttest means was significant at the .05 level. The critical t was 2.04 and the calculated t was 2.76. The null hypothesis was rejected and the research hypothesis was accepted (see Appendix D).

Table 2

t Test Results of the Differences Between Pretest and Posttest Mean Scores

Group	N	$\bar{X}$	s	t
Control	18	8.33	10.91	2.76*
Experimental	18	21.67	17.35	

N: Number of students in the group

X: Mean score

s: Standard deviation

t: Test statistics

*: Significant at .05 level

It was concluded that there was a significant difference between those second grade students who received daily measurement lessons and those who received daily measurement lessons involving the use of estimation skills. Those second grade students who received measurement instruction plus experiential estimation activities scored significantly higher.

Observations

The experimental group was very excited about the experiential estimation activities. Each child had an

opportunity to develop estimation skills through hands on activities daily. It became a challenge to them to see how accurate they could be with their estimations. Competition built up among children to see who could get closest to the exact answer. The students looked forward to their daily lessons. They had very thoughtful looks on their faces during the posttest.

The control group enjoyed their experiential activities also and learned to measure well. They looked forward to their daily lessons in measurement. They each had daily opportunities to interact with materials. When the posttest was given, however, they were very disappointed that they were not prepared for it. They had very quizzical looks on their faces during the testing and asked if they would get a grade on it.

Conclusions and Discussion

The findings of this study support the findings of experts and organizations involved in the field of mathematics education such as, Reys and Reys, Keller and Shannahan, Bestgen, Office of Educational Research and Improvement, and National Council of Teachers of Mathematics. Young children can be taught estimation

skills in measurement. Instruction in estimation results in improved problem solving skills and can further some key thinking processes. Making direct estimates of number is the most effective way of developing early number sense. Providing young children with experiential activities in measurement estimation is an effective way to teach estimation. Hands on involvement increases math achievement and problem solving skills.

Estimation needs to be taught in schools because it is used in 80% of the real-world problem solving situations outside the classroom. Children need to be prepared for real world situations.

Due to the increasing use of calculators and computers to do mathematical calculations the use of good estimation skills is vital to test effectively the reasonableness of answers. Students should be prepared in school to function effectively in the real world and to solve problems effectively.

The study went well. If done again, nothing would be changed except, that time and temperature measurement would be included. Due to the fact that the classes involved had already studied these two concepts, they had to be omitted.

Limitations

Interpretation of the results of this investigation is limited by the following constraints:

1. The number of students was small.
2. The study was limited to 24 days.

Implications for Future Research

Further studies are needed in this area. Studies should include a larger sample drawn from various socio-economic backgrounds to be applicable to the general population of second grade students. Experiential studies should be done involving other concepts such as number, addition, and subtraction to see if estimation in these areas can be taught to young children. It is hoped that the experiential approach to solving estimation problems will be included in the mathematics curriculums and textbooks for young children. The young child learns most effectively through first hand experiences. The experiential lessons and manipulative materials to go with the lessons should be provided by textbook companies.

Implications for Classroom Practice

Schools should place special emphasis on teaching and testing concepts involving estimation. Schools tend to teach what is tested on standardized tests, so standardized tests should include estimation items.

Schools should place great importance on the teaching and testing of estimation, since a large portion of our real-world problems involve estimation. Children need to be taught to think and solve problems in our world. The skill of estimating is an important concept to teach because it answers both of these needs.

Estimation needs to be included in curriculums for teaching mathematics. Estimation skills are essential and must be given high priority within every school program. Few mathematics topics provide the wealth of benefits, both immediate and long term, as does estimation. Parents need to be made aware of the importance of estimating, so they can emphasize it when they are working with their children.

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APPENDIX A

INSTRUMENTS

Pretest

1. Do you think it is longer around your ankle or your waist?
2. Display a balance scale, a glue bottle, and an interlocking cube. How many interlocking cubes will it take to balance the glue bottle?
3. Display a small sheet of construction paper and a playing card. How many playing cards will it take to cover the sheet of construction paper?
4. How many meters long is the movie screen?
5. How many centimeters wide is a sheet of notebook paper? Display a sheet of notebook paper.
6. How many meters wide is our bulletin board?
7. How many centimeters long is a large piece of construction paper? Display a large sheet of construction paper.
8. Display a liter container, a half liter container, and a quarter liter container. Label the containers A, B, C. Display a one-eighth liter cup. If eight people come to a birthday party, which container will provide a cupful of lemonade for each person present?

9. Hold up two Social Studies books, a box of 16 crayons and a pencil sharpener. Which object weighs about a kilogram?
10. Hold up a second grade tablet, a pecan and a math book. Which one weighs about a gram?

Posttest

1. Do you think it is longer around your head or your waist?
2. Display a balance scale, a chalkboard eraser and a crayon. How many crayons will it take to balance the eraser?
3. Display a four centimeter square of construction paper and a large sheet of construction paper. How many small squares will it take to cover the large sheet of construction paper?
4. Display a meter stick. How many meters high is the classroom door?
5. Display a centimeter stick. How many centimeters long is a one dollar bill?
6. How many meters wide is our blackboard?
7. How many centimeters is it around your spelling book?
8. Display a liter container and a one-eighth liter container. How many one-eighth liter cups will it take to fill the liter container?
9. Display two science books, an eraser, and a pencil. Which object weighs about a kilogram?

10. Display an apple, a grapefruit, and a nickel.
    Which object weighs about a gram?

These tests have been deemed valid by two experts in the field of education.

Name	Date
_____	_____
_____	_____

APPENDIX B

LESSON PLANS

Objective of the Project

The children will be able to solve problems involving measurement mathematics.

Lesson Plan Day One

Experimental Group:

"Hold up a book and ask, "How many toothpicks long do you think the book is?" Invite several responses before a volunteer actually measures the book with toothpicks. Display another object and repeat the process. This time demonstrate how to first record the estimate and then the measure on the blackboard similar to line master 63 (see Appendix C). Repeat this activity until children appear familiar with the process of recording the estimate and actual measurement for each object they consider. Provide each child with a copy of Line Master 63. Invite the children to continue to estimate and measure in a unit of their choice such as toothpicks, paper clips, or straws. Have them record their discoveries on Line Master 63" (Coombs, Harcourt, Travis, & Wannamaker, 1967, p. 161).

Control Group:

Group the children in pairs. Give each enough adding machine tape for each child to measure with. Have a child lie down on the floor while the other child unrolls the adding machine tape the length of his or her partner. Ensure that the roll is in a straight line from the heels to the top of the head. Tell the child to mark the partner's height on the tape with a pencil. The strip is then cut and the process is repeated for the second child. Have the children print their names on the tapes.

Provide children with toothpicks, straws, and popsicle sticks. Explain that they will discover how tall they are using these materials. Demonstrate that they must place the units end to end so that they touch but do not overlap on their height tape. Remind the children that they must place the first item so that it touches the end of the strip. After they have placed their units along their strips, have them count the units and record on "My Height" chart (see Appendix C).

Lesson Plan Day Two

Experimental Group:

Ask the children to make measuring tapes using one inch squares of paper, beans, and toothpicks (see Appendix C). Make them different lengths. Use a different material to create three different tapes. Review with children the necessity of having each unit placed end to end. Have them print their name on the back of each measuring tape they make. Have the children use each tape measure to measure three different classroom objects. Use Line Master 63 for this purpose (see Appendix C). Have the children print on the recording sheet the name of the object, the name of the unit used and an estimate of the number of units before actually measuring. Repeat this procedure for each of the three objects measured (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Give each child either a wooden spoon, a spatula, a candle, a comb, or a table knife to measure. Have them trace the shape of it on a large piece of construction paper. Have them use popsicle sticks, toothpicks, and beans to measure the object. Each

unit should be lined up along the object, pasted down, counted and recorded. Trace the object again before each measurement. Use a different piece of construction paper for each. Have them record a sentence or two about their project (For example, "My comb is as long as eight toothpicks.") When they are finished, ask individuals to show their work and describe how long their objects are. Display on bulletin board (Randolph, Westley, 1987).

Lesson Plan Day Three

Experimental Group:

Display a collection of objects ranging from about 1 cm to 20 cm. Provide each child with a ruler and introduce the centimeter as another measuring unit. Discuss the difference in the sizes of objects that would be measured with centimeters and meters. Display meter stick during discussion. Let the children examine their rulers. Let them measure the tips of their fingers to see which finger is approximately one cm in width, how many fingers are 2 centimeters, and what is the measure of each finger and thumb. Have the children point to the 10 cm marking.

Select an object (pencil) and ask, "Do you think this pencil is longer or shorter than 10 cm?" Demonstrate how to measure it with a ruler. Emphasize the alignment of the end of a pencil and the 0 markings on the ruler. Say "The pencil is 15 cm long. Is it longer or shorter than 10 cm?" Place the pencil on the appropriate sheet of paper labelled "longer" or "shorter". Have volunteers repeat the process for at least two more objects. Give each child two sheets of paper. Have them label a sheet "shorter" and the other sheet "longer". Have the children measure their glue bottle, scissors, crayon, straw, and toothpick. Sort objects onto corresponding sheets of paper by tracing objects on paper.

Control Group:

Give each child a centimeter ruler. Let them examine carefully. Discuss the difference in objects that would be measured with a centimeter ruler and a meter stick. Display meter stick during discussion. Have them measure each finger tip to see how many centimeters long each is. Have them trace, measure and record measurement for their glue bottle, scissors, crayon, a straw and a toothpick.

Lesson Plan Day Four

Experimental Group:

Provide each child with a copy of Practice Supplement page 96 (see Appendix C) from the Addison-Wesley Mathematics Book 2. Have them read and follow the directions for estimating and measuring objects. Have volunteers share their answers while others compare and check their answers.

Control Group:

Provide each child with a copy of Practice Supplement 95 (see Appendix C) from the Addison-Wesley Mathematics Book 2. Have children read and follow directions for measuring objects to the nearest centimeter. Have volunteers give answers while others compare and check their own answers.

Lesson Plan Day Five

Experimental Group:

Give each child three construction paper shapes. Have them first estimate each side and record then, actually measure each side and record to determine perimeter of shapes. Compare estimates to actual

measurements. Let volunteers discuss their estimates and actual measurements while others compare and check theirs.

Control Group:

Give each child three construction paper shapes. Have them use centimeter rulers to measure each side. Record each measurement and add to find the perimeter of each shape.

Lesson Plan Day Six

Experimental Group:

Provide each child with three floor plans whose dimensions are an even number of square centimeters (see Appendix C). Provide one-inch centimeter tiles cut from construction paper for each child. Ask the children to first estimate the number of tiles it will take to cover the floor and record on paper. Cover floors with tiles. Count, record and compare to estimate (Randolph, Westley, 1987).

Control Group:

Group children in groups of four. Let them play concentration using shapes cut from one-centimeter graph paper (see Appendix C). They are to find two

shapes that have the same area. The player turns over the two shapes and counts the number of squares in each. If the number is the same, the player keeps the shapes. When all the pairs of shapes have been found, the players count their shapes. The winner is the player with the greatest number of shapes (Randolph, Westley, 1987).

Lesson Plan Day Seven

Experimental Group:

Introduce the meter measure. Display a meter stick. Select a child to look for something in the room that is a meter long or wide. Give the child the meter stick to check out his or her estimate. Ask, "Is it one meter long?" "Is it shorter or longer than one meter?" Record item measured on chart (see Appendix C). Invite other volunteers to estimate which classroom objects are about one meter. Have each volunteer check out his or her estimate. Record each discovery on the chart (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Introduce the meter stick. Display a meter stick.

Provide pairs of children with a meter stick or a string the length of a meter stick. Have them work together to measure two objects in the room that could appropriately be measured with a meter stick. Some examples are the door, blackboard, length of room, distance across the room, and the teacher's desk. Have children record measurements. Measurements can be shared and compared later.

Lesson Plan Day Eight

Experimental Group:

Mark off several meters with the children on the playground. Show them how to draw a mark at the end of the meter stick, then move the stick so that zero is at the mark. Mark the end of the ruler again, move the ruler, and so on. Connect the marks with a straight line. Have each child in turn stand at the zero mark and throw a bean bag as far as he or she can. Have each child estimate how far he or she threw the bean bag. Measure with a meter stick to see how accurate his estimate was (Randolph,Westley, 1987).

Control Group:

Have children mark off a number of meters on the

playground. Show them how to draw a mark at the end of the meter stick, then move the stick so that zero is at the mark. Mark the end of the stick again, move the stick, and so on. Have the children label the marks with numbers. Then ask each child in turn to stand at the zero mark and throw a bean bag as far as he or she can. Tell the children to record the number of meters the bean bag was thrown. When everyone has had a turn, have them try to beat their own record (Randolph, Westly, 1987).

Lesson Plan Day Nine

Experimental Group:

Show two jars of different capacities, one filled with orange juice and the other with grape juice. Ask the children which jar they think holds more juice. Have them estimate the number of ice cube cups that can be filled with each jar of juice. Have children pour each jar of juice into a different ice cube tray and count the cups that are filled. Have a child place a number card beside each jar to show how many cups were filled. Have another child place the more and less labels next to the jars to show which holds more and which holds less. Repeat the activity with

several other pairs of jars filled with fruit juices. Make sure the children have a chance to compare tall, narrow containers with short, broad containers. This will help them as they move toward understanding that taller containers do not necessarily hold more. End the activity by putting the ice cube trays in the freezer. When the popsicles have hardened slightly, place a popsicle stick in each ice cube cup. Serve the popsicles for snack the following day (Randolph, Westley, 1987).

Control Group:

Guide small groups of children to various work areas to freely explore the capacity of various sizes and shapes. Encourage children to pour, empty, and fill containers in as many ways as possible. Circulate among the groups to observe and encourage discussion. Ask questions like these: "How did you fill this container?"; "Which container can you fill up quickest? Why?";"Is this container full? How do you know?" (Coombs, Harcourt, Travis, Wannamaker, 1987).

Lesson Plan Day Ten

Experimental Group:

Have children work in groups of four. Provide

containers of different sizes and a pan of sand to use in experimenting with pouring. The children estimate which containers hold the same amount, then measure and place the ones that hold the same amount on attached pairs of paper plates (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Let the children work in groups of four. Place at least three bowls, three cups, and three spoons of different sizes at a center. Children compare the capacity of each type of object and place the ones that hold the most on the mat to create a "Big Meal" (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Lesson Plan Day Eleven

Experimental Group:

Have mugs of different shapes and sizes available for the children to choose from. Let each child choose a mug. Ask, "How could we make sure everyone has the same amount of lemonade?" Once all reasonable responses have been discussed, designate a small paper cup as the sharing measure. Pour out cupfuls of lemonade and pass it to each child in turn, asking

them to pour it into their mug. Once all children have one paper cup of lemonade in their mug, ask each child to guess how many cupfuls of lemonade their mug will hold. After the third cupful has been poured into each mug, ask the children to compare their amounts with the person next to them. Remind the children that fair sharing means everyone having the same amount of lemonade, which is three paper cups full.

Observe the children's reaction to the level of lemonade in their mug. Key questions at this point would be: "How much lemonade is in your mug?"; "Do you have the same amount of lemonade as the person next to you?"; "Does it look like the same amount? Why not?" (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Provide a bucketful of sand and four different size containers for groups of four children. Provide a plastic cup. The children pour sand into the containers to find out which container holds 20 cups (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Lesson Plan Day Twelve

Experimental Group:

"Work with children to make a measuring jar. Place a piece of masking tape from the base to the top of a one-quart jar. Have a child fill a small paper cup "to the brim" with water (Colored with food dye) from the pitcher or bucket and pour its contents into the quart jar. Have him or her mark the level of water in the quart jar and write a one next to the mark. Call on another child to put one more cupful of water in the jar and mark the new level. Continue in the same way until the jar is full, each time marking the water level."

"Pour out the water from the measuring jar and show the children some other jars of various shapes and sizes. The jars should be filled with water (colored with food dye) and have capacities less than one liter. Have the children select a jar and estimate how high the water level will be if its contents is poured into the measuring jar. Have someone pour and tell how much water the jar holds. Encourage the children to use terminology such as about three marks high, a little more than two marks high, or between

four and five marks high. Repeat with the other jars and label them with the measurements" (Randolph, Westley, p. 45, 1987).

Control Group:

"Work with the children to make a measuring jar. Place a piece of masking tape from the base to the top of a one-quart jar. Have a child fill a small paper cup "to the brim" with water (colored with food dye) from the pitcher or bucket and pour its contents into the quart jar. Have him or her mark the level of water in the quart jar and write a one next to the mark. Call on another child to put one more cupful of water in the jar and mark the new level. Continue in the same way until the jar is full, each time marking the water level."

"Set up a lemonade stand on a table in the classroom and explain that each level on the measuring jar is worth one cent. Have the children come to the stand one at a time and tell how much lemonade they want to buy. Measure the amount requested by the first child, pour it into a cup, and give it to the child to drink. Then have that child run the lemonade stand for the next child, and so on until everyone has

had a turn "buying" and "selling" lemonade" (Randolph, Westley, p. 45, 1987).

Lesson Plan Day Thirteen

Experimental Group:

Have children work in groups of four. Provide a pan of sand, a marked liter container, a collection of other containers, three hoops and three cards to label each hoop as more, less, and same. Have the children estimate whether or not the container will hold more, less, or the same as a liter, then actually measure with sand to check their estimates. Record answers on blackline master 63 (see Appendix C) (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Provide a pan of sand, a marker, a liter container, a collection of other containers, three loops of yarn and three cards to label each loop as more, less, or same. Have the children working in groups of four sort the containers by comparing them to the specially marked liter container. Record answers on blank paper (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Lesson Plan Day Fourteen

Experimental Group:

Display a collection of liter containers of different sizes and shapes and a paper cup. Choose one of the containers and ask, "How many cupfuls do you think it will take to fill this liter container?" Invite children to offer estimates. Ask a volunteer to fill the liter container with cups of water, as another volunteer snaps an interlocking cube for each cup added. Ask, "Was your estimate too high? Was it too low?" Invite children in groups of four to investigate the capacity of a liter container following the procedure of filling it with a ladle, cup or smaller container, and placing a cube for each measure added. After the children have investigated the capacity of a liter container using at least two different measuring units by first estimating and then measuring, invite them to share the findings with the larger group. Have them place their towers of cubes with the measuring unit they used on large pieces of construction paper labelled with the name of the measuring unit (see Appendix C).

Control Group:

Display a liter container, two half liter containers, four one fourth liter containers, and eight one eighth liter containers. Using a container of water, demonstrate how many half liters, fourth liters, and eighth liters it takes to fill a liter container. Direct children to discover these findings with the same materials in groups of four.

Lesson Plan Day Fifteen

Experimental Group:

Display a collection of objects of varying mass. Select two objects which obviously have different masses, such as a rubber ball and a large seashell. Ask, "Which do you think is heavier?" Invite the children to guess which object is heavier and then pass the two objects around for each child to hold. After all children have held the objects say, "Now that you've held the objects, do you have a different idea of which object is heavier?" Display a balance scale. Point to each pan as you ask, "What do you think will happen when we put the ball here and the seashell here? Why?" Place an object in each pan of the balance. Ask, "Why do you think one pan of the

balance is hanging lower?" Switch the objects so that they are in different pans on the balance scale. Ask, "Which pan do you think will hang lower this time? Why?" Choose two objects that are similar but of different masses (a dictionary and a storybook). Ask the children to estimate which object is heavier. Pass around the books for all to hold. Before the books are placed on the balance scale, provide an opportunity for the children to change their estimates. Repeat the process asking children to estimate, hold, and measure (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Divide children in small groups of five or six. At each work area, place a balance scale. Guide children to explore the mass of classroom objects such as crayons, pencils, chalkboard erasers, books, and other objects. At the end of the exploration, bring the children together. Encourage them to share and describe their discoveries by asking "Which object was heaviest, which was lightest, which objects weighed the same?" (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Lesson Plan Day Sixteen

Experimental Group:

Display a balance scale, a collection of classroom objects and nonstandard units for measuring mass such as popsicle sticks, bottle caps, coins, or cubes. Select a child to choose an object and place it in one of the balance scale pans. Then challenge a volunteer to "balance" the balance, using a non-standard unit such as popsicle sticks in the other pan. When the scale is balanced, have the child count the number of popsicle sticks and announce, "The mass of the glue bottle is the same as 14 popsicle sticks," or whatever it turns out to be.

Holding up another classroom object say, "You found that the glue bottle has the same mass as 14 popsicle sticks, or whatever, how many popsicle sticks do you think we will need to balance the scissors?" Volunteers in turn hold the glue bottle in one hand and the scissors in the other, offer an estimate, and then measure the mass as a group. Have children work in small groups or with partners to estimate and then check the mass of various objects against popsicle sticks. The children will make a quick record of

their estimates and discoveries on Line Master 63 (see Appendix C) (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Control Group:

Divide children in small groups of five or six. At each work area, place a balance scale, two loops of string, cards labelled lighter and heavier, a collection of classroom objects, and an object marked in some special way (an eraser with a sticker on it, a glue bottle with a picture of a story character or some other object). Have them sort the objects according to whether they are lighter or heavier than the marked reference object, after comparing them on the balance scale. Record on sheet of paper (Coombs, Harcourt, Travis, & Wannamaker, 1987).

Lesson Plan Day Seventeen

Experimental Group:

Display a kilogram mass. Pass it around so that the children can feel the weight of it. Have the children get into groups of five or six. At each work area place a balance scale, three loops of string and three cards labelled lighter, same, and heavier, a collection of classroom objects, and a kilogram mass.

Have the children first sort the objects into the three loops by estimating their weight compared to the kilogram mass, then check their estimates using the balance scale. Objects should be resorted as necessary. Record discoveries on a chart (see Appendix C). Discuss accuracy of estimates.

Control Group:

Display several kilogram masses. Introduce the kilogram measure as a standard unit of measurement by showing the samples. Ask, "Why do you think people find the mass of things using kilograms, rather than the other kinds of things we have used?" Discuss the unreliability of the units of things we have used thus far.

Working in small groups, children are to make kilogram masses by using the balance scale. Place a kilogram mass on one side and place enough modeling clay on the other side to make a kilogram mass. Place in display center (Coombs, Harcourt, Travis, & Wannamaker).

Lesson Plan Day Eighteen

Experimental Group:

Display a collection of gram weights. Pass around and let children feel the weight. Allow children to form small groups of five or six. At each work station, provide a balance scale, three loops of yarn labelled lighter, heavier, and same, and several objects for weighing. First sort objects into loops by estimating weight, then check estimates by weighing on balance scale against the gram mass. Record results on chart (see Appendix C).

Control Group:

Display several gram masses. Ask, "Why do you think people find the mass of things using grams, rather than the kinds of things we have used?" Discuss the unreliability of the units of measurement they have used thus far. Working in small groups, children are to use sand, plastic sandwich bags, and twisters to make gram masses. Provide a balance scale and a gram mass at each work station.

Review

A review of the previous lesson plans will be done using chapter nine on measurement in the Addison-Wesley Mathematics Book Two. The instructor will explain the directions and ask for questions prior to the children doing the workbook pages.

Review Day Nineteen

Experimental Group:

Read and follow directions for pages 225, 226, and practice supplement 93. Allow volunteers to give answers while others check their responses.

Control Group:

Read and follow directions for pages 225, 226, and practice supplement 93. Allow volunteers to give answers while others check their responses.

Review Day Twenty

Experimental Group:

Read and follow directions for pages 227, 228, and practice supplement 94. Allow volunteers to give answers while others check their responses, after children complete the pages.

Control Group:

Read and follow directions for pages 227, 228, and practice supplement 94. Allow volunteers to give answers while the others check their responses after children complete the pages.

Review Day Twenty-One

Experimental Group:

Read and follow directions for pages 229, 230, and enrichment supplement 69. After children have completed the pages, allow volunteers to give answers while the others check their responses.

Control Group:

Read and follow directions for pages 229, 230, and practice supplement 95. After children have completed the pages, allow volunteers to give answers while the others check their responses.

Review Day Twenty-Two

Experimental Group:

Read and follow directions for pages 231, 232, and practice supplement 96. After children have

completed the pages, allow volunteers to give answers while the others check their responses.

Control Group:

Read and follow directions for Reteaching Supplement 70 and Enrichment Supplement 70. After children have completed the pages, allow volunteers to give answers while the others check their responses.

Review Day Twenty-Three

Experimental Group:

Read and follow directions for pages 233, 234, and Practice Supplement 97. After children have completed the pages, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for pages 233, 234, and Practice Supplement 97. After children have completed the pages, allow volunteers to give answers while the others check responses.

Review Day Twenty-Four

Experimental Group:

Read and follow directions for pages 235, 236,

and Practice Master 98. After children have completed the pages, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for pages 237, 238, and Parctice Master 99. After children have completed the pages, allow volunteers to give answers while the others check responses.

Review Day Twenty-Five

Experimental Group:

Read and follow directions for pages 237, 238, and Practice Master 99. After children have completed the pages, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for Reteaching Supplement 72, and Enrichment Supplement 72. After children have completed the pages, allow volunteers to give answers while the others check responses.

Review Day Twenty-Six

Experimental Group:

Read and follow directions for pages 239 and 240. After children have completed the pages, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for pages 239 and 240. After children have completed the pages, allow volunteers to give answers while the others check responses.

Review Day Twenty-Seven

Experimental Group:

Read and follow directions for pages 241 and 242. After children have completed the pages, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for pages 241 and 242. After children have completed the pages, allow volunteers to give answers while the others check responses.

Review Day Twenty-Eight

Experimental Group:

Read and follow directions for page 243. After children have completed the page, allow volunteers to give answers while the others check responses.

Control Group:

Read and follow directions for page 243. After children have completed the page, allow volunteers to give answers while the others check responses.

Following each review, ask for and discuss any questions the children may have.

APPENDIX C

SUPPORTIVE MATERIALS

I am measuring	I am using	I estimate	I measure

Line Master 63

Object Measured	Measurement in meters

My Height Chart

Name _____ 92

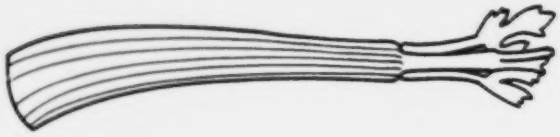
I am _____ toothpicks high.

I am _____ straws high.

I am _____ popsicle sticks high.

Name _____ To follow text page 232

First guess each length. Then measure with your centimeter ruler.

1. 

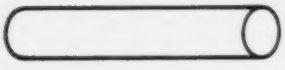
Your estimate _____

Your measure _____

2.

Your estimate _____

Your measure _____

3. 

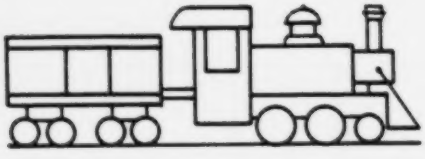
Your estimate _____

Your measure _____

4.

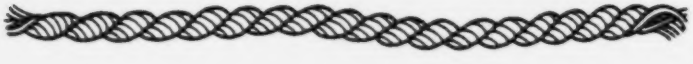
Your estimate _____

Your measure _____

5. 

Your estimate _____

Your measure _____

6. 

Your estimate _____

Your measure _____

7.

Your estimate _____

Your measure _____

8.

Your estimate _____

Your measure _____

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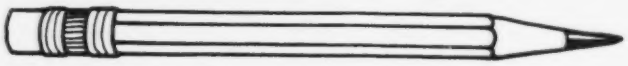
Name _____

To follow text page 230

Use your centimeter ruler. Give each length to the nearest centimeter.

1. 

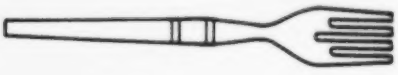
_____
Nearest centimeter

2.

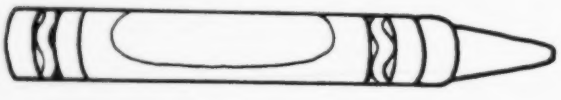
_____
Nearest centimeter

3.

_____
Nearest centimeter

4. 

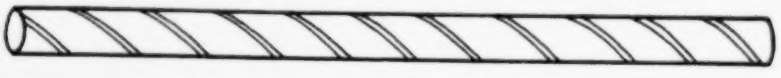
_____
Nearest centimeter

5. 

_____
Nearest centimeter

6.

_____
Nearest centimeter

7. 

_____
Nearest centimeter

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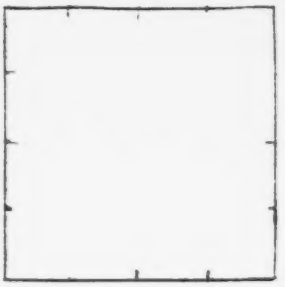
Floor Plans

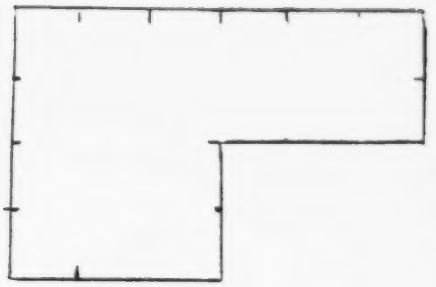
I estimate ____ squ. cm.

I measure ____ squ. cm.



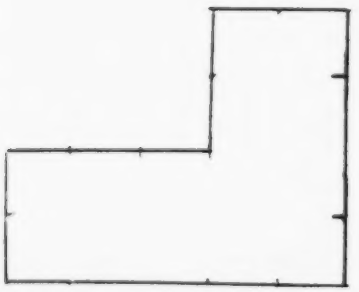
I estimate ____ squ. cm.

I measure ____ squ. cm.

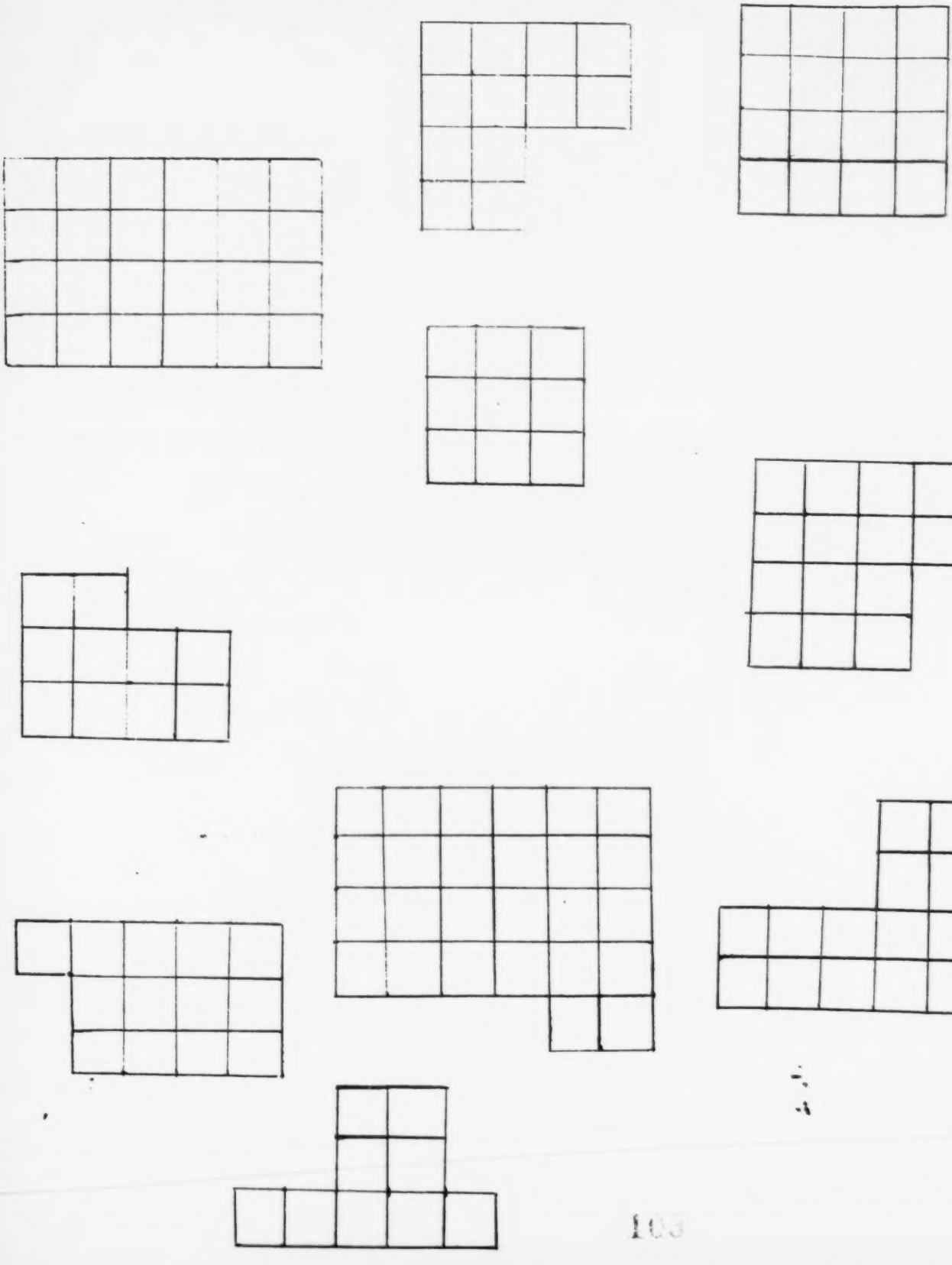


I estimate ____ squ. cm.

I measure ____ squ. cm.



Concentration Cards



APPENDIX D

STATISTICAL DATA

Control Group

Pretest Scores	Posttest Scores	Difference
20	50	30
30	60	30
40	50	10
20	30	10
40	60	20
30	60	30
50	60	10
60	70	10
50	60	10
40	40	0
40	10	-30
50	50	0
10	10	0
40	50	10
40	50	10
40	50	10
40	30	-10
50	50	0

$\bar{X}_2 = 8.33$

$S_2 = 10.91$

Experimental Group

Pretest Scores	Posttest Scores	Difference
60	70	10
70	80	10
50	70	20
10	70	60
90	70	-20
50	50	0
80	80	0
40	70	30
40	70	30
60	80	20
30	60	30
30	70	40
50	50	0
40	80	40
20	50	30
30	80	50
60	70	10
40	70	30

$\bar{X}_1 = 21.67$

$S_1 = 17.35$

t Test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2} \quad \frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{21.67 - 8.33}{\sqrt{\frac{(18-1)(17.35)^2 + (18-1)(10.91)^2}{18 + 18 - 2} \quad \frac{1}{18} + \frac{1}{18}}}$$

$$t = \frac{13.34}{\sqrt{\frac{(17)(301.02) + (17)(119.03)}{34} \quad \frac{2}{18}}}$$

$$t = \frac{13.34}{\sqrt{\frac{(5117.34) + (2023.51)}{34} \quad \frac{1}{9}}}$$

$$t = \frac{13.34}{\sqrt{\frac{7140.85}{34} \quad \frac{1}{9}}}$$

$$t = \frac{13.34}{\sqrt{(210.03) \quad \frac{1}{9}}}$$

$$t = \frac{13.34}{\sqrt{(210.03)\ \frac{1}{9}}}$$

$$t = \frac{13.34}{\sqrt{(23.34)}}$$

$$t = \frac{13.34}{4.83}$$

$$t = 2.76$$

Calculated $\underline{t}$ = 2.76

Critical $\underline{t}$ = 2.04

The calculated $\underline{t}$ is greater than the critical $\underline{t}$.